

# ENDOGENOUS GROUP FORMATION AND MONITORING

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## **Abstract**

*This paper is an attempt to explore the determinants behind various group formations. We show that the group formation depends on the efficiency of information transmission. In our point of view, the optimal grouping structure should minimize the distortion induced by the informative imperfection so that it is easiest to sustain the cooperative equilibrium.*

**Keywords:** *Grouping structure, Peer Monitoring, Intermediary, Information, Repeated Game*

## **Introduction**

In the real world, there exist various economic, ethnic and tribal divisions. For example, ancient Greece once was a collection of poleis. Ancient Rome divided its territory into different provinces. And Chinese political division has experienced the transition from system of Enfeoffment to Province. One may wonder what determinants work behind so many different grouping structures. Unfortunately, we cannot directly find the answer from the existing economic theories. Only a few papers have mentioned this issue. Dixit (2002), for example, once pointed out that the consideration of endogenous structure may be an extension to his model. But he didn't give the specific analysis and further explanation. Obviously, grouping structure is an important but neglected topic.

This paper is an attempt to explore the diversification of grouping structures by an infinitely repeated matching game. In our opinion, the optimal group division structure should minimize the distortion induced by the informational imperfection. Here we assume that the members within a group own inside information, who can disclose the miscreant actions she observed to show her innocence. Besides, there is also public information which can be observed by a trustworthy intermediary. These two types of information constitute the information transmission mechanism. The specific grouping structures will determine the efficiency of peer monitoring and public signal transmission. Since the rule of information transmission mechanism varies across different situation, it requires different optimal grouping structures.

This paper is organized as follows. In Section 2, we review the relating literatures. In Section 3, we build a model by assuming that the payoff might vary each period. Under this condition, we find that the information transmission mechanism does affect the formation of grouping structure. In Section 4 we conclude.

## **Literature Review**

Now let's explain how our work is connected with existing models to the notion of stereotype. The first one is matching game theory which has rapidly developed since the pioneering work of Rosenthal (1979). Kandori (1992) once built a two-group matching game, in which each player can only be matched with another one from the opposite group. Ellison (1994)

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loosed this constraint and extended the model to a randomly matching game, which convention is followed by our model. The literature referring to grouping problem includes Genicot and Ray (2003) analyzed the insurance market structure; Lester(2005) discussed the trade-off between increased scale benefit and monitoring deficit based on Dixit(2003)etc. All of their works can be regarded as direct inspiration to us.

Secondly, we assume that there is an informative intermediary, based on the work of Milgrom et al (1990), Greif et al (1994), Greif (1989), (1992)-(1994). The intermediary receives and sends out signals. Besides, following Banerjee et al (1994), Besly and Coate(1995), Aghion and Gollier(2000) and Che and Yoo(2001), we also allow peer monitoring to work in our model. Both peer monitoring and public signal constitute information transmission mechanism.

Furthermore, we establish a model of a repeated game based on the folk theorem’s logic. After the work of Friedman (1971), Auman and Sharpley (1976), Rubinstein (1979), Fudenberg and Maskin (1986) systematically concluded folk theorem with perfect monitoring. On the other hand, the imperfect (public) monitoring folk theorem was developed by Green and Porter(1984), Abreu Pearce and Stachatti (1990) and Fudenberg, Levine and Maskin (1994), etc. Following the above ideas, our model in this paper belongs to imperfect monitoring with public and inside signals.

### Endogenous group formation and monitoring

In this section, we demonstrate the role of grouping structure by a repeated game with discrete time and infinite horizon. The population of players is  $2N$  which can be freely divided into  $2N/n$  groups<sup>1</sup>. At the beginning of each period, players are randomly matched into pairs. After the matching, the players choose to work or shirk. The player cannot discern the identity of her opponent in each period. Moreover, players cannot distinguish the two types of stage games in history since there are two different types of payoff matrices: with probability  $1-z$ , the two matched players will face a stage game of prisoner’s dilemma shown by Figure 1; with probability  $z$ , they will get  $h_t$  independent of their actions<sup>2</sup>. The random variable  $h_t$  follows a distribution of  $F$  with a density function  $f$  from  $w$  to  $+\infty$ .

		Player 2	
		Work	Shirk
Player 1	Work	$w, w$	$-h_t, h_t$
	Shirk	$h_t, -h_t$	$0, 0$

Figure 1

In each period, if one player shirked, with probability  $x\alpha(n)$ , the other members within her group can observe the miscreant action. Suppose there is a monitoring intermediary, which will receive and send out the signals. Each player can freely decide to disclose the miscreant action to the intermediary after she observes it. Afterwards, if only one member in a group shirked, then with probability  $y\beta(n)$  which is independent of  $x\alpha(n)$  given  $n$ , the intermediary can observe

<sup>1</sup> Once the grouping structure is formed, it cannot change during the process of the matching game.

<sup>2</sup> This can be regarded as a noise.

the action directly. Here  $x, y \in [0, 1]$ . They are independent of the structure design  $n$ , standing for the current level of the current monitoring technology. And  $\alpha(n), \beta(n)$  are the variables affected by the grouping structure. To simplify, we suppose neither players nor the intermediary can lie. At the end of each period, the intermediary declares all the results it has observed.

Assume that neither  $\alpha$  nor  $\beta$  is zero. Moreover, for any  $n \in S$ , the p.d.f. of  $h$ ,  $f(\cdot)$  satisfies:  $f(w) > \frac{(1-\delta)}{(1-z)\delta} / [1 - (1-x\alpha)(1-y\beta)]$ . According to the above assumption there is at least one fixed point in the interval  $[w, \infty)$ , which satisfies the following equation:

$$h(n) = w + \frac{(1-z)\delta w}{1-\delta} \int_w^{h(n)} f(h) dh [1 - (1-x\alpha)(1-y\beta)].$$

Intuitively, the larger the value of  $h$  is, the harder it is to achieve cooperation in the stage game<sup>3</sup>. Define  $\bar{h}$  to be the largest fixed point, which can be regarded as the largest scope to sustain a cooperative equilibrium.

**Proposition 1.** For any  $x$ , there is a value  $\underline{y}(x)$ . When  $y \leq \underline{y}(x)$ , the optimal structure  $n^* = \arg \max \alpha(n)$ . Similarly, for any  $y$ , there is a value  $\underline{x}(y)$ . When  $x \leq \underline{x}(y)$ , the optimal structure  $n^* = \arg \max \beta(n)$ .

An intuitive explanation of Proposition 1 is that if the between-group information is too blurred, the optimal structure will be decided by how to optimize the within-group information. On the contrary, if the within-group information is too unclear, the optimal structure will be decided by how to optimize the between-group information. According to the above statements, we can also get the following result.

**Proposition 2.** The optimal solution  $n^*$  is only determined by  $\alpha, \beta, x$  and  $y$ .

From the above proposition we can see that there might be various structures due to different  $\alpha, \beta, x$  and  $y$  under different circumstances. Hence there might appear a monopoly, oligarch or competitive market across various industries.

## Conclusion

In this paper, we analyze the optimal structure arrangement with a repeated game model, where the payoff is not fixed in each period. We investigate the effect from the within- and between-group technology. Moreover, we also find that the optimal structure is determined by the monitoring technique and grouping structure. Such results can explain why there are so many different grouping structure in the real world.

The extension to this paper might be the following direction. 1) Endogenizing the matching rule instead of randomization following the work of Ghosh et al (1996). 2) Considering the adverse selection problem to discuss how the heterogenous individuals would form the optimal structure.

<sup>3</sup> Formal explanation is given by Proposition 2.

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