R&D AND OUTPUT BEHAVIOR OF DUOPOLISTIC FIRMS WITH CONJECTURAL VARIATIONS IN R&D[*]

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Abstract
We analyze R&D and output behavior of oligopolistic firms under R&D spillover when they possess conjectural variations in R&D. By use of the conjectural variations approach, the preceding researches on R&D are extended to more general analysis and the behavior of actual firms in a lot of industries are more elucidated. This paper shows that the results on the R&D and output behavior are totally reversed by whether firms act in an aggressive or a passive way in the R&D decision. It is demonstrated that the conventional results hold under a range of competition or collusion as well. CVs in R&D play a crucial role in determining the level of technological improvement.

Keywords: *R&D, Spillover, Conjectural Variations, Oligopolistic Firms*

Introduction

Innovation plays an indispensable role in firm activities such as production, sales, new product development, and rivalry. Now innovation conducted by a firm is classified into two types: One type is process innovation, which aims to reduce the production costs by improving the production and/or sales process; and the other is product innovation, which aims to enhance the quality of a product and develop new products. d'Aspremont and Jacquemin (1988) focus on oligopolistic firms that conduct research and development (R&D) aiming at process innovation, and investigate their R&D and output behavior when spillovers on the fruits of R&D activities exist. It is shown that with high spillover rates, the R&D investment of each firm increases when the firms cooperate at both stages, in comparison with those under R&D cooperation and output competition and under R&D and output competition, while, with low spillover rates, they obtain a different result from the above.

By introducing product differentiation and using the more general model, Kamien et al. (1992) extend the analysis of d'Aspremont and Jacquemin (1988), and consider the effects of the establishment of an R&D research joint venture (RJV) under both R&D competition and R&D cartelization, demonstrating that its establishment increases the R&D under R&D cartelization in comparison with the one under R&D competition. It is also shown that RJV cartelization leads to the maximum welfare (social surplus) within the four cases of R&D competition, R&D cartelization, RJV competition, and RJV cartelization. The influence of strategic R&D and collusion on the market performance has been investigated by many researchers (e.g., Simpson and Vonortas (1994)) in addition to the work.[1] The role of R&D investment as the strategic (business stealing) method is widely considered in trade theory and trade policy as well.

Gollop and Roberts (1979), Iwata (1974), and Suzuki et al. (1993) demonstrate by empirical analysis that oligopolistic firms in several industries possess conjectural variations in

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[1] There is, for example, Symeonidis (2003) as recent research concerning product innovation.

quantities. Their results explicitly differ from the traditional Cournot assumption. This causes us to easily predict that real firms employ conjectural variasions when choosing the levels of R&D.

Ohashi and Haruna (2009) shed light on output and R&D behavior of oligopolistic firms under various competition in the second stage by incorporating conjectural variations in quantities. Their approach using conjectural variations in R&D can make it possible for us to grasp such behavior under various rivalry conditions in an output market apart from Cournot competition, Bertrand competition, and Collusion.[2, 3]

They show that the R&D and output behavior of firms is explicitly affected by the degree of quantity competition (which is measured by such conjectural variations): That is, given large (small) R&D spillovers, positive conjectural variations lead to larger (smaller) R&D investment than no conjectural ones. This extends the analyses of d'Aspremont and Jacquemin (1988), and Kamien et al. (1992).

However, no papers address R&D and quantity behavior of firms with conjectural variations in R&D who predict their rivals' reactions to their decision on R&D, when making their decisions. Firms in the automobile, liquid crystal panel and semiconductor industries almost certainly make a decision on R&D investment, anticipating their rivals' reactions on its decision. In these industries conjectural variations in R&D are recently taking an important role more and more to well manage firms. Therefore, the conventional idea that they determine their R&D strategy, ignoring their rivals' response like the Cournot assumption, is right irrelevant. Then we incorporate conjectural variations in R&D into our analysis. By this our analysis could generalize and reconsider traditional analyses such as, for example, d'Aspremont and Jacquemin (1988), and Kamien et al. (1992), and throws a new real light on the investigation of the R&D and output behavior of oligopolistic firms.

We obtain the following results from the analysis. Whether conjectural variations in R&D are positive or negative has a great impact on the decisions of firms as well as the rates of spillovers. Specifically, the firms make a greater (less) investment in R&D in the presence of positive (negative) conjectural variations than in zero conjectural variations. Like this, the level of market performance is reversed by whether the conjectural variations are positive or negative. This implies that the assumption of zero conjectural variations will lead to misinterpretation on firm behavior and market performance. The validity of d'Aspremont and Jacquemin's result (1988) is also confirmed in an extended model.

The remainder of the paper is organized as follows: In Section 2 we provide a duopolistic two-stage game model with conjectural variations in R&D; and in Section 3 we take two

[2] To use such a static model for the analysis of oligopolistic firms is, therefore, maintained to be inappropriate as "most real world oligopoly interaction takes place over a number of periods. In this sense, static oligopoly models are of limited use" (Cabral (2000, p. 32)). It is stated that to use dynamic models for the analysis of such oligopolistic firms can resolve a methodological and conceptually justifiable problem encountered in a static model (see, e.g., Riordan (1985)). Traditional single-stage game models of oligopoly with and without conjectural variations in quantities are posited as a static model that does not explicitly consider a reaction of each rival.

[3] Nevertheless, many significant results relating to oligopolistic firms are derived from the use of static models. In addition, as asserted by Dixit (1986), Martin (1993), and Shapiro (1989), analysis using a static model with conjectural variations has the following advantages. It allows us to capture firm behavior under various degrees of competition in a unified and simple model, and since conjectural variations are a convenient way of parameterizing oligopolistic behavior, they are useful for comparative static purposes. Moreover, it is easier to derive explicit results within a static model than within a dynamic model. Some advantage will be displayed when we consider firm behavior with the use of multistage game models with R&D as a strategic variable: Namely, incorporating conjectural variations into an oligopolistic model enables us to elucidate the behavior of firms. For example, Eaton and Grossman (1986) analyze the behavior of exporting firms and optimal policy in oligopolies with conjectural variations.

scenarios, depending on whether or not such conjectural variations exist, and then examine how the conjectural variations influence R&D and output of firms with R&D spillovers. In Section 4 we compare prices, profits of firms, and welfare. Moreover, to promote comprehension of the relationship of profits with the degrees of spillovers and competition we provide an illustration by the use of an example of numerical computation. The introduction of CVs in R&D provides the relationship between R&D and continuous changes, not discrete changes, in R&D rivalry. The final section concludes the paper

R&D and Output Behavior of Duopolistic Firms with Conjectural Variations in R&D

1. The Model

We elucidate R&D and output behavior of oligopolistic firms with conjectural variations (CVs) in R&D and reexamine some of previous results on firm R&D strategy. Following the two-stage game model of d'Aspremont and Jacquemin (1988), we employ a duopoly model in which two firms simultaneously choose the levels of R&D at the first stage and then outputs at the second stage.[4] In the second stage they are always involved in Cournot quantity competition.

The market's linear inverse demand function takes the form of

$$p = a - b(q_i + q_j),\ \ a > 0 \ \text{ and } \ b > 0,\ \ i, j = 1, 2,\ \ i \neq j, \tag{1}$$

where q_i and p denote the output of firm i and output price, respectively. Both firms produce a homogeneous good.

When the firms invest in R&D to make an innovation in the process of manufacturing and/or sales, the cost function of firm i is given by $c_i = c - x_i - \rho x_j, i \neq j,$ where c is marginal costs, x_i and x_j are cost reductions acquired by firms i and j as a result of their R&D investments, respectively, and $\rho\ (\in [0,1])$ is a spillover rate.[5] The presence of spillovers implies that firms cannot make the fruits of their R&D investments appropriable perfectly except for $\rho = 0$.[6] Put it differently, some fruits of each firm obtained by its own R&D activities flow out to other firms in the same industry without payment, so firm i's R&D lowers not only its own production costs but also those of its rival. Particularly, $\rho = 1$ means that all of firms perfectly share information on all results obtained by their R&D activities each other. When they establish a research joint venture (RJV), the information on the results is perfectly shared among them. In order to reduce its production costs by x_i firm i has to spend $vx_i^2/2, v > 0,$ as R&D costs, where v stands for the efficiency or productivity of R&D. R&D investment exhibits diminishing-returns-to-scale.

The overall profits of firm i are given as

[4] Salant and Shaffer (1998, 1999) demonstrate that, given the assumption that firms are symmetric ex ante, the symmetric solution is deduced as the R&D strategy of each firm for their joint profit maximization; however, this solution is not optimal but suboptimal, i.e., dominated by the asymmetric solution.

[5] Spillovers are in R&D results like d'Aspremont and Jacquemin (1988), different from Kamien et al. (1992).

[6] Incidentally, Bernstein and Nadiri (1991, p. 22) state from the empirical research on six industries of the U.S. that "A 1% increase in R&D spillovers causes a range of variable cost reduction from 0.05% to 0.24%."

$$\pi_i = (p - c_i)q_i - \frac{v}{2}x_i^2 = [a - b(q_i + q_j) - c + x_i + \rho x_j]q_i - \frac{v}{2}x_i^2, \quad i, j = 1, 2, \quad i \neq j. \tag{2}$$

Given x_i and x_j, the firms choose outputs at the second stage so as to maximize their own profits. Differentiating (2) with respect to q_i, we obtain the first-order condition for profit maximization of firm *i*:

$$\frac{\partial \pi_i}{\partial q_i} = a - 2bq_i - bq_j - c + x_i + \rho x_j = 0, \quad i \neq j. \tag{3}$$

Its second-order condition is given by $\partial^2 \pi_i / \partial q_i^2 = -2b < 0$. The quantity equilibrium in the second stage is locally stable, i.e., $D = (\partial^2 \pi_i / \partial q_i^2)(\partial^2 \pi_j / \partial q_j^2) - (\partial^2 \pi_i / \partial q_j \partial q_i)(\partial^2 \pi_j / \partial q_i \partial q_j) = 3b^2 > 0.$

By solving (3), we obtain the outputs for given R&D levels:

$$q_i = \frac{(a - c) + (2 - \rho)x_i + (2\rho - 1)x_j}{3b}. \tag{4}$$

Furthermore, we obtain the industry output:

$$Q = q_i + q_j = \frac{2(a - c) + (1 + \rho)(x_i + x_j)}{3b}. \tag{5}$$

When we substitute (4) and (5) into (2), the overall profits of each firm are given as

$$\pi_i = bq_i^2 - \frac{v}{2}x_i^2, \quad i = 1, 2. \tag{6}$$

Next, we turn to R&D decisions of both firms at the first stage. The Cournot assumption that each firm behaves as if its rivals will not alter their outputs to a change in the output of the former, however, does not go well with the behavior of real firms. Rather, they have CVs in quantities (see, e.g. Iwata (1974) and Suzuki et al. (1993)). [7] Similarly, it is easily inferred that lots of firms decide their levels of R&D investment, predicting rivals' reactions to a change in their R&D investments, that is, they possess CVs in R&D investment as in quantities, although the CVs have been ignored in the previous researches. Among others, as such CVs play an important role in decisions of real firms, we incorporate them into the analysis. Thus firms *i* and *j* choose their own R&D, employing their own CVs. Differentiating (6) with respect to x_i yields the first-order condition:

[7] Ohashi and Haruna (2009) consider the R&D and output behavior of firms without the Cournot assumption.

$$\frac{\partial \pi_i}{\partial x_i} = \frac{2[2-\rho+\beta_i(2\rho-1)][(a-c)+(2-\rho)x_i+(2\rho-1)x_j]}{9b} - vx_i = 0,$$
$$i, j = 1, 2, \quad i \neq j, \qquad (7)$$

where $\beta_i = \partial x_j / \partial x_i$ and $\beta_j = \partial x_i / \partial x_j$ are the CVs in R&D of firms *i* and *j*, respectively. Now β_i is the rate of R&D change of firm *j* anticipated by firm *i* in response to a change in its own R&D, and is assumed to be constant. Although in the previous literature CVs in R&D have been set to $\beta_i = \beta_j = 0$ until now, we assume that their CVs lie in the ranges $\beta_i \in [-1, 1]$ and $\beta_j \in [-1, 1]$. A positive and large value of CVs captures predictions that rivals will respond aggressively to any attempt by firm *i* to increase R&D investment. Then such predictions inversely lead firm *i* to less aggressive behavior, i.e., small R&D investment. In the end, the anticipation of aggressive responses of the rivals gives rise to more collusive equilibrium in the first stage. As the relationship between the value of CVs in R&D and firm behavior in the first stage the following one is established: Both firms competitively determine their R&D by Cournot-type behavior when $\beta_i = \beta_j = 0$, perfect-competitively determine their R&D by Bertrand-type behavior when $\beta_i = \beta_j = -1$, and determine their R&D by collusive behavior when $\beta_i = \beta_j = 1$, that is firms form an R&D cartel in which they choose their R&D levels so as to maximize the sum of their overall profits.[8] Parameter β_i is, therefore, considered to be an index denoting the degree of competition or collusion among firms in the first stage.[9] Incidentally, the conjectural variations in R&D may or may not be consistent, because we have no special interest in the consistent CVs.

The R&D reaction function of firm *i* is reduced to:

$$x_i = -\frac{2(2\rho-1)[2-\rho+\beta_i(2\rho-1)]}{2(2-\rho)[2-\rho+\beta_i(2\rho-1)]-9bv} x_j - \frac{2(a-c)[2-\rho+\beta_i(2\rho-1)]}{2(2-\rho)[2-\rho+\beta_i(2\rho-1)]-9bv}.$$

This function is a function obtained under various scenarios ranging from much more competition than Cournot-type competition to full cooperation (cartelization) in R&D, as shown by the value of CVs. It is dependent on the slopes of the reactions curves whether their curves cross the "wrong" way.[10] It is helpful to use a numerical example to check the slopes. According

[8] See, for example, Shapiro (1989), Martin (1993), and Dixit (1986), for the relationship between the value of CVs in quantities and the degree of competition or collusion in output markets. When the value is zero, the definition of R&D competition is the same as that, i.e. pure competition, in, e.g., d'Aspremont and Jacqumin (1988), Kamien et al. (1992), and Amir and Wooders (1998).

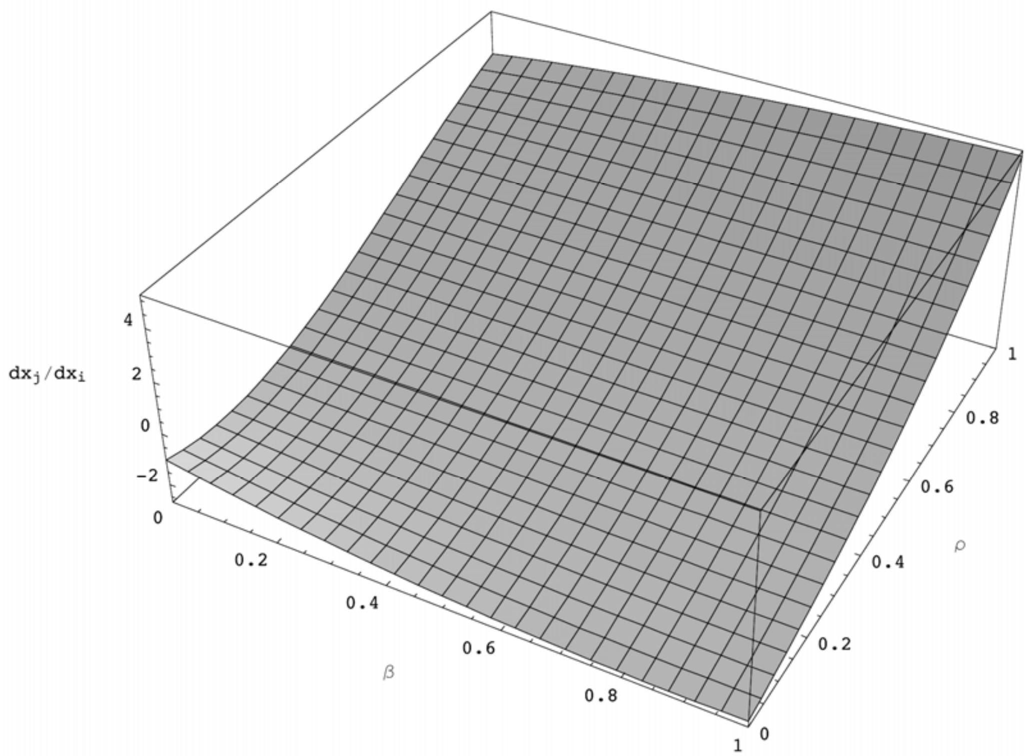
[9] Previous studies by d'Aspremont and Jacquemin (1988), Kamien et al. (1992), and so on do not consider R&D and output behavior of firms involved in much more competition than Cournot-type competition in the R&D stage.

[10] The slope of firm *i*'s R&D reaction function on the plane (x_i, x_j) is given by:

$$\frac{dx_j}{dx_i} = -\frac{2(2-\rho)[2-\rho+\beta_i(2\rho-1)]-9bv}{2(2\rho-1)[2-\rho+\beta_i(2\rho-1)]}.$$

Then this is of either sign: That is, the reaction curves may or may not slope downward, as first pointed out by Henrigues (1990). Amir and Wooders (1998, p.63) "shows that under R&D competition the firms' reaction functions may cross the "wrong" way." In order for CVs to be consistent, the slopes of the reaction curves of firms to the example with $b = 0.5$ and $v = 0.25$, the slopes are affected more strongly by spillovers rather than by CVs. In particular, the signs of the slopes turn from positive to negative signs as the spillovers approach one. The slope of the firm *i*'s reaction curve is positive and maximized at the point of $(\beta, \rho) = (-1, 0)$, but negative and minimized at the point of $(\beta, \rho) = (1, 1)$. Further, there is doubtless some range where the curves cross the wrong way. The example indicates that the slope of the R&D reaction curve gets positive like quantity reaction curves and increasingly great as the value of the CVs approach -1. To see the pure effect of a change in the value of CVs on the reaction curve of firm *i* we set $\rho = 0$ as a simple case, so increased CVs shift its reaction curve downward. However, its effect will get more complicated when spillovers exist.

Figure 1. Slope of the reaction function in R & D space



The term $(2\rho - 1)x_j$ in the numeration of (7) represents the effect of firm *j*'s R&D investment on firm *i*'s costs through spillovers (say the free-rider effect). If spillover rates are high, i.e., $1/2 < \rho \leq 1$, then the R&D of firm *j* lowers both its own and firm *i*'s production costs,

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i and *j* have to be equal to β_i and β_j, i.e., $dx_j / dx_i = \beta_i$ have to hold. Specifically, when there exist CVs in R&D, the consistent CVs are obtained when $\beta_i = (2-\rho)[1 \pm \sqrt{(2\rho + 7 - 36bv)/(2\rho - 1)}\,]/2$. When there are not spillovers, the CVs are reduced to $\beta_i = 1 \pm \sqrt{36bv - 7}$.

which in turn increases their profits. In contrast, if the rates are low, i.e., $0 \le \rho < 1/2$, then firm *j* can gain an advantage over firm *i* by its R&D and depress the latter. Meanwhile, parts $\beta_i(2\rho - 1)$ in $[2 - \rho + \beta_i(2\rho - 1)]$ in the numeration of (7) include the effect of the CVs on the R&D. Now the combined externality of both effects may be positive, negative, or zero: It is positive when $\beta_i > 0$ and the spillover rates are high, while negative when $\beta_i < 0$ and the rates are low. When the externality is positive (negative), CVs have an incentive (a disincentive) for both firms to invest more (less) in R&D.

Hereafter, let us assume that the two firms are symmetric, i.e., $\beta_i = \beta_j = \beta$, and $x_i = x_j = x$ for simplicity. Then the symmetric conjectural variations equilibrium (SCVE) is established in the two-stage game model of duopoly. By using this assumption and solving (7), we obtain the value of R&D investment under the SCVE:

$$x^* = \frac{2[2 - \rho + \beta(2\rho - 1)](a - c)}{\Delta}, \tag{8}$$

where $\Delta = 9bv - 2(1 + \rho)[2 - \rho + \beta(2\rho - 1)]$. The second-order condition in the first stage is assumed to be satisfied.[11] Whether or not the SCVE is stable depends on such parameters as R&D conjectural variations, spillover rates, and R&D efficiency.[12]

Substituting (8) into (4), (5), and (6) yields the outputs and profits under the SCVE as follows:

$$q^* = \frac{3v(a - c)}{\Delta}, \quad Q^* = \frac{6v(a - c)}{\Delta} = 2q^*,$$
$$\pi^* = b\left[\frac{3v(a - c)}{\Delta}\right]^2 - \frac{v}{2}\left[\frac{2[2 - \rho + \beta(2\rho - 1)](a - c)}{\Delta}\right]^2.$$

Then consumer and producer surplus, and welfare under the SCVE are given as

$$CS^* = \frac{18bv^2(a - c)^2}{\Delta^2} = 2b(q^*)^2, \quad PS^* = \Pi^* = 2\pi^*,$$

[11] The second-order condition for firm *i* is given by:

$$\partial^2 \pi_i / \partial x_i^2 = 2[2 - \rho + \beta(2\rho - 1)]^2 / 9b - v < 0.$$

If this is satisfied, then there exists an internal solution in the R&D stage. As a simple case, given $\rho = 0$, it is reduced to $9bv > (2 - \beta)^2$, while, given $\beta = 0$, reduced to $9bv > (2 - \rho)^2$.

[12] It follows that the SCVE has an internal equilibrium and is stable for the pairs of (bv, ρ) such as $S_1 = \{(bv, \rho) \mid 2(2 - \beta)/3 \le bv \text{ and } 0 \le \rho \le 1\}$, $S_2 = \{(bv, \rho) \mid 1/2 \le bv < 2(2 - \beta)/3$ and $[3(1 - \beta) - \sqrt{(1 + \beta)^2 + 6bv(1 - 2\beta)}\,]/2(1 - 2\beta) < \rho \le 1\}$, and $S_3 = \{(bv, \rho) \mid 4(1 + \beta)/9 \le bv < 1/2$ and $[(1 + \beta) + 3\sqrt{(1 + \beta)^2 - 2bv(1 - 2\beta)}\,]/2(1 - 2\beta) < \rho \le 1\}$. Henriques (1990) and Haruna (2003) examine the local stability of the symmetric R&D equilibrium in the first stage. Besides, Amir and Wooders (1998) consider the stability of the symmetric equilibrium under R&D competition in the first stage and Cournot quantity competition in the second stage.

$$W^* = CS^* + PS^* = \frac{2v\{18bv - 2[2-\rho+\beta(2\rho-1)]^2\}(a-c)^2}{\Delta^2}, \tag{9}$$

where Π^* stands for the sum of the overall profits of the duopolistic firms.

We take two cases according to CVs in R&D: One is Case 1 ($\beta \neq 0$) where each of the firms chooses its R&D, anticipating its rival's reaction to a change in its R&D; and the other is the benchmark case ($\beta = 0$) where it chooses its R&D, anticipating its rival's no reaction to its change.

The amounts of R&D investments in Case 1 and the benchmark case are given, respectively, as

$$\hat{x} = \frac{2[2-\rho+\beta(2\rho-1)](a-c)}{\hat{\Delta}}, \tag{10}$$

$$\tilde{x} = \frac{2(2-\rho)(a-c)}{\tilde{\Delta}}, \tag{11}$$

where $\hat{\Delta} = 9bv - 2(1+\rho)[2-\rho+\beta(2\rho-1)]$ and $\tilde{\Delta} = 9bv - 2(1+\rho)(2-\rho)$.

Let us consider how R&D investment changes as parameters β and ρ change. First, from the comparative statics on CVs, $d\hat{x}/d\beta = 18bv(a-c)/(2\rho-1)/\hat{\Delta}^2$ is yielded from (10). This shows that the effect on R&D levels depends crucially on spillover levels: That is, an increased value of a firm's CVs about its rival's reaction leads to a reduction in its R&D investment if the levels are low, i.e. $0 \leq \rho \leq 1/2$, while it leads to its increase if the rates are high. Given small (great) spillover rates, the effect of reducing costs (the free-rider effect) caused by R&D spillovers which is beneficial to its rival is small (great), so that an increased value of the CVs causes its marginal revenue in R&D to decrease (increase), which in turn increases (decreases) its R&D. Put it differently, note that the less competitive or more collusive the first stage, the more likely firms are to decrease (increase) their R&D for small (great) spillovers.

Second, with respect to the effect of spillover rates on R&D in Case 1 we have

$$\frac{d\hat{x}}{d\rho} = \frac{2(a-c)}{\hat{\Delta}^2}\{-9bv + 2[2-\rho+\beta(2\rho-1)]^2 + 9bv(2\beta)\}.$$

In the conventional case with zero CVs, their effect is reduced to $d\tilde{x}/d\rho = 2(a-c)[-9bv + 2(2-\rho)^2]/\tilde{\Delta}^2$, so it gets negative as long as the first stage SCVE is stable. Generally, $d\hat{x}/d\rho$ is of either sign, but the sufficient condition for $d\hat{x}/d\rho > 0$ is $\beta \geq 1/2$. In this case increased spillover rates lead to an increase in R&D. In contrast, when the SCVE is stable and CVs are negative, the increased ones cause R&D to reduce.

2. Comparisons of R&D investments and Outputs

In order to see how CVs in R&D plays a role in the R&D decision of duopolistic firms we compare the R&D investments $\hat{x}$ and $\tilde{x}$ under the SCVE in both Case 1 $(\beta \neq 0)$ and the benchmark case $(\beta = 0)$, respectively. Using (10) and (11), we obtain the following result:

$$\hat{x} - \tilde{x} = \frac{18\beta bv(2\rho - 1)(a-c)}{\hat{\Delta}\tilde{\Delta}}. \tag{12}$$

We compare both R&D investments. From (12), (i) given $\beta > 0$, we have $\hat{x} \leq \tilde{x}$ for $0 \leq \rho \leq 1/2$ (equality holding at $\rho = 1/2$) and $\hat{x} > \tilde{x}$ for $1/2 < \rho \leq 1$, and, (ii) given $\beta < 0$, we have $\hat{x} \geq \tilde{x}$ for $0 \leq \rho \leq 1/2$ (equality holding at $\rho = 1/2$) and $\hat{x} < \tilde{x}$ for $1/2 < \rho \leq 1$ as well.

We summarize these results as the following proposition:

Proposition 1. (i) Suppose that firms have non-negative conjectural variations in R&D, i.e., $0 \leq \beta \leq 1$. Then the equilibrium R&D investments $\hat{x}$ and $\tilde{x}$ satisfy the following:

If $0 \leq \rho \leq 1/2$, then $\hat{x} \leq \tilde{x}$ (equality holding at $\rho = 1/2$), and if $1/2 < \rho \leq 1$, then $\tilde{x} < \hat{x}$.

(ii) Suppose that firms have negative conjectural variations in R&D, i.e., $-1 \leq \beta < 0$, the equilibrium R&D investments satisfy the following:

If $0 \leq \rho \leq 1/2$, then $\tilde{x} \leq \hat{x}$ (equality holding at $\rho = 1/2$), and if $1/2 < \rho \leq 1$, then $\hat{x} < \tilde{x}$.

The outcome of Proposition 1 shows that the classification of R&D levels is substantially affected by CVs in R&D as well as spillovers. The result (i) of the proposition is intuitively explained as follows. With positive CVs, a firm conversely behaves in a passive way, not in an aggressive way, when choosing R&D, because its rival is anticipated to respond to its choice in an aggressive way. Besides, the magnitude of the benefit due to a reduction in production costs, which the rival could obtain as a result of the firm's R&D, is small, that is large R&D is beneficial to the firm itself, but detrimental to the rival if spillover rates are low. Thus the firm with no CVs chooses to prefer larger R&D for low spillovers. On the other hand, the result (ii) of the proposition is explained as follows. With negative CVs, a firm behaves in an aggressive way in the decision of R&D because its rival is anticipated to respond to its decision in a passive way. Taking account of the free-rider effect caused by R&D spillovers, firms prefer greater and smaller R&D according to lower and higher spillover rates, respectively.

Incidentally, when firms establish an RJV ($\rho = 1$), whether they have an incentive to increase more or less R&D investment is dependent of whether they have positive or negative CVs: Namely, positive CVs lead to higher R&D, but negative CVs to lower R&D in comparison with its level under no CVs. In contrast, if there are no spillovers, R&D is depressed by positive CVs, but encouraged by negative CVs.

We see that the results (i) and (ii) on the R&D classification are totally the reversed by whether or not CVs is positive. The classification depends critically on whether firms act in an aggressive or a passive way based on the anticipated reactions of their rivals. This implies that not only spillovers but also CVs in R&D substantially affect the progress of technological improvement. The effectiveness of R&D policy relies greatly on firm CVs as well as spillovers as well. To improve its effectiveness the government has to take account of the values of the CVs of firms in each industry.

d'Aspremont and Jacquemin (1988) compare two R&D investments under the cases where duopolistic firms act either non-cooperatively (equivalent to $\beta = 0$) or full-collusively (equivalent to $\beta = 1$) in the first stage and act non-cooperatively in the second stage. As shown above, their result explicitly carries over to the intermediate case $(0 < \beta < 1)$ where they act less non-cooperatively and less-collusively. This means that their result with respect to the R&D classification is robust independent of whether firms fully or partially cooperate in the R&D decision.

Let us compare the outputs $\hat{q}$ and $\tilde{q}$ which are represented as a function of R&D investment as follows:

$$\hat{q} = \frac{a - c + (1+\rho)\hat{x}}{3b}, \quad \text{and} \quad \tilde{q} = \frac{a - c + (1+\rho)\tilde{x}}{3b}$$

Then we have

$$\hat{q} - \tilde{q} = \frac{(1+\rho)(\hat{x} - \tilde{x})}{3b}. \tag{13}$$

From the results of Proposition 1 and (13) we immediately establish one of the following proposition:

Proposition 2. (i) Suppose that firms have non-negative conjectural variations in R&D, i.e., $0 \le \beta \le 1$. Then the equilibrium outputs $\hat{q}$ and $\tilde{q}$ satisfy the following: If $0 \le \rho \le 1/2$, then $\hat{q} \le \tilde{q}$ (equality holding at $\rho = 1/2$), and if $1/2 < \rho \le 1$, then $\tilde{q} < \hat{q}$.

(ii) Suppose that firms have negative conjectural variations in R&D, i.e., $-1 \le \beta < 0$, then the outputs satisfy the following:

If $0 \le \rho \le 1/2$, then $\tilde{q} \le \hat{q}$ (equality holding at $\rho = 1/2$), and if $1/2 < \rho \le 1$, then $\hat{q} < \tilde{q}$.

As demonstrated by this proposition, each firm produces less output in the presence of positive CVs than in their absence when spillover rates are low, while it produces more output in the presence of them when the rates are high. We provide an intuitive explanation for the outcome of the proposition. We recall from (4) that whenever spillover rates are higher (lower) than 0.5, CVs have the effect to increase (decrease) R&D, and its rival's R&D as well as its own one, furthermore, increases (decreases) output for positive CVs, as shown in (12), while the reverse result holds for negative CVs. Thus, given high (low) spillovers, the outputs are larger (smaller) when they have positive CVs than when they do not, while the inverse result holds in the case of negative CVs.[13]

The result of d'Aspremont and Jacquemin (1988), which is derived under non CVs, is the same result as our result. Like the result of R&D comparison their result on quantity carries over to the more general case.

[13] As known from (3), the quantity reaction curves of both firms are downwardly sloping. As an increase in positive CVs deceases (increases) R&D investment as long as spillovers are small (large), this causes the reaction curves and then the second-stage symmetric equilibrium to shift downward (upward). With respect to negative CVs the reverse story holds.

3. Comparisons of Prices, Profits, and Welfare

Since the inverse demand function is given by (1), we immediately obtain the following results concerning the price ranking from Proposition 2.

Corollary. (i) Suppose that firms possess non-negative conjectural variations in R&D, i.e., $0 \le \beta \le 1$. Then the equilibrium prices $\hat{p}$ and $\tilde{p}$ satisfy the following relationship:

If $0 \le \rho \le 1/2$, then $\hat{p} \ge \tilde{p}$ (equality holding at $\rho = 1/2$), and if $1/2 < \rho \le 1$, then $\tilde{p} > \hat{p}$.

(ii) Suppose that firms possess negative conjectural variations in R&D, i.e., $-1 \le \beta < 0$. Then the prices satisfy the following relationship:

If $0 \le \rho \le 1/2$, then $\tilde{p} > \hat{p}$ (equality holding at $\rho = 1/2$), and if $1/2 < \rho \le 1$, then $\hat{p} > \tilde{p}$.

This corollary is obtained directly from Proposition 2 (i) and states that, given small spillovers, consumer surplus is smaller in the benchmark case than in Case 1 with positive CVs, but, given large spillovers, the opposite result is derived. The results (i) and (ii) show that the magnitude of consumer surplus is totally reversed by whether CVs are positive or negative. From the proposition, even if there are no spillovers, consumer surplus gets less (larger) as a result of positive (negative) CVs rather than in the absence of them, while if firms organize an RJV, the surplus gets larger (less) as a result of positive (negative) CVs. This implies that it is detrimental to the consumer that firms possess positive CVs, while it is beneficial that they possess negative CVs. In general, it depends the value of CVs on whether R&D promotion policy is beneficial to the consumer. The latter result (i) of Corollary is the same as the results of Kamien et al. (1992) and d'Aspremont and Jacquemin (1988). It shows that their results hold in the extended model with various degrees of R&D competition.

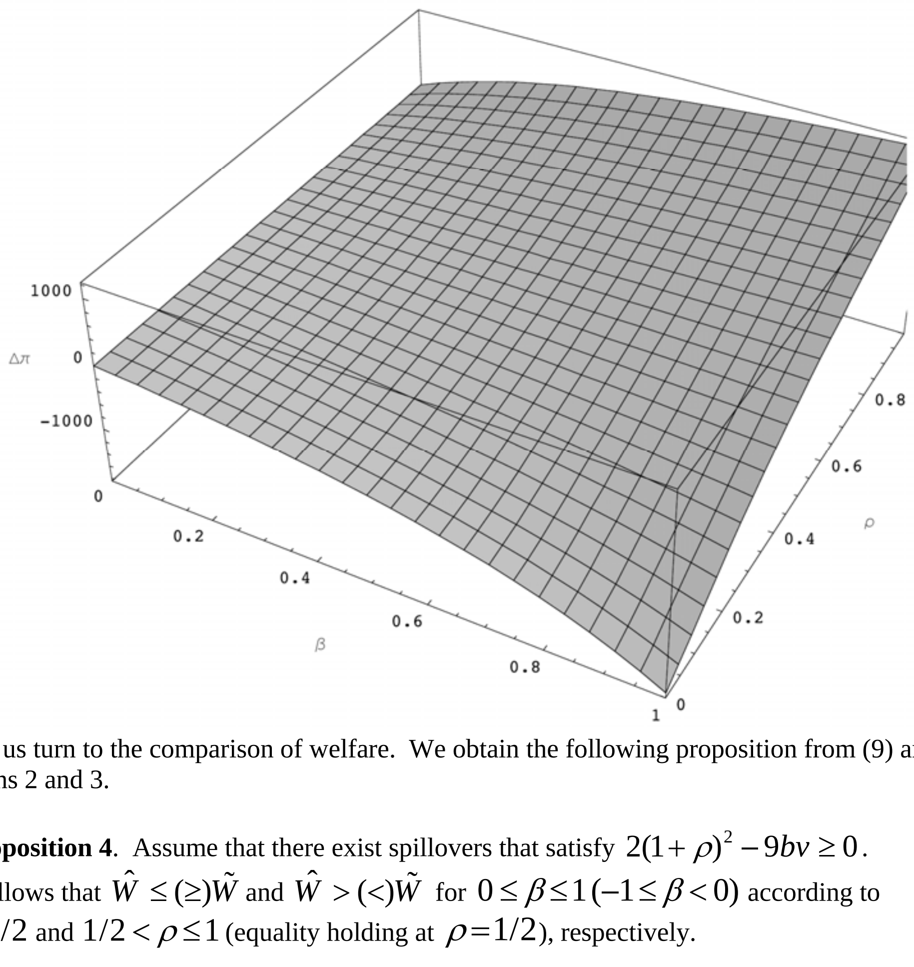
Now we make a comparison of the firms' overall profits. Then it follows that
$\hat{\pi} - \tilde{\pi} = b(\hat{q}^2 - \tilde{q}^2) - v(\hat{x}^2 - \tilde{x}^2)/2 =$
$(\hat{x} - \tilde{x})[4(1+\rho)(a-c) + (2(1+\rho)^2 - 9bv)(\hat{x} + \tilde{x})]/18b$, where
$\hat{q} + \tilde{q} = [2(a-c) + (1+\rho)(\hat{x} + \tilde{x})]$. We establish the following proposition.

Proposition 3. Assume that there exist spillovers that satisfy $2(1+\rho)^2 - 9bv \ge 0$. Then it follows that $\text{sgn}(\hat{\pi} - \tilde{\pi}) = \text{sgn}(\hat{x} - \tilde{x})$ unambiguously holds.

Taking Proposition 1 into consideration, we can derive some more specified results as follows: When there exist spillovers that satisfy $2(1+\rho)^2 - 9bv \ge 0$, we obtain that $\hat{\pi} \le (>) \tilde{\pi}$ for $0 \le \beta \le 1\,(-1 \le \beta < 0)$ (equality holding at $\rho = 1/2$) and $\hat{\pi} > (<) \tilde{\pi}$ for $0 \le \beta \le 1\,(-1 \le \beta < 0)$ according to $0 \le \rho \le 1/2$ and $1/2 < \rho \le 1$, respectively. The proposition shows that firm profits are explicitly affected by CVs in R&D which firms possess before choosing R&D levels. The CVs play an important role in determining their choice. Alternatively, we notice that the result of Kamien et al. (1992), that the profits of firms under R&D collusion are larger than those under R&D competition, is also valid under partial competition or cooperation in R&D. The results on profit comparison are reversed by whether or not CVs are positive. It might not be easy for both the stability condition of the SCVE in the first stage and the condition $2(1+\rho)^2 - 9bv \geq 0$ to be satisfied simultaneously.

To get an intuitive comprehension on the comparison, we employ a numerical computation with $a-c=40, b=0.5, v=0.5$ and $0 \leq \beta \leq 1$. Figure 1, where $\Delta\pi = \hat{\pi} - \tilde{\pi}$, demonstrates the relationship of $\Delta\pi$ to (β, ρ). According to it, as CVs and spillover rates both rise, a difference between $\hat{\pi}$ and $\tilde{\pi}$, i.e. $\Delta\pi$, becomes larger. On the other hand, if the rates are small, then $\Delta\pi$ decreases and becomes negative as the CVs rise. Among others, $\Delta\pi$ is maximized at $(\beta, \rho) = (1,1)$, but minimized at $(\beta, \rho) = (1,0)$.

Figure 2 Comparison of Profits in Case 1 and the Benchmark Case



Let us turn to the comparison of welfare. We obtain the following proposition from (9) and Propositions 2 and 3.

Proposition 4. Assume that there exist spillovers that satisfy $2(1+\rho)^2 - 9bv \geq 0$. Then, it follows that $\hat{W} \leq (\geq) \tilde{W}$ and $\hat{W} > (<) \tilde{W}$ for $0 \leq \beta \leq 1$ $(-1 \leq \beta < 0)$ according to $0 \leq \rho \leq 1/2$ and $1/2 < \rho \leq 1$ (equality holding at $\rho = 1/2$), respectively.

We see that the magnitude of welfare explicitly depends on both values of spillovers and CVs.[14] For example, when spillovers are low (high), it is larger when firms act in an aggressive way in the R&D decision stage $(-1 \le \beta < 0)$ than when they do not $(\beta = 0)$, while the reverse result is derived when they act in a passive way. Kamien et al. (1992) show that welfare (total surplus) is maximized in the equilibrium under RJV cartelization, where corresponds to the case with $(\beta, \rho) = (1, 1)$. However, they do not consider R&D levels under intermediate cases between R&D competition (including RJV competition) and R&D cartelization (including RJV cartelization) and under more intense competitive cases than R&D Cournot competition. It also depends on spillover rates whether both the collusive equilibrium and the most aggressive firm behavior in the R&D decision stage lead to less or more welfare than the Cournot equilibrium $(\beta = 0)$. The level of welfare is substantially affected by the anticipation of a firm on its rival's response to a change in its R&D. This result points out that although their roles have been traditionally largely overlooked, the government should not overlook the roles of CVs in welfare if it pursues the R&D policy to improve welfare.

Conclusions

We have investigated how CVs in R&D influence R&D and output behavior of duopolistic firms by employing a two-stage game model. It has been conventionally assumed that a firm makes its output decision, anticipating that its rival takes no response to a change in its output like the Cournot assumption, whereas it had been empirically already elucidated by lots of papers that actual firms behave differently from the assumption when choosing their outputs. We can easily infer that the same argument as this also holds true in their R&D decisions. Thus the use of CVs in R&D as the tool to investigate the behavior of real firms in more detail is informative.

We have shown that the R&D and output choices of duopolistic firms are greatly affected by whether or not they possess CVs. For example, when the levels of R&D and output in the presence of CVs are compared with those in their absence, their classifications are totally reversed as the value of CVs changes, say, positive to negative. As demonstrated by our results, disregard of CVs in R&D will result in a misunderstanding about firm behavior and the government R&D policy.

In previous papers (e.g. d'Aspremont and Jacquemin (1988) and Kamien et. al (1992)) results on R&D classification are obtained under comparison of the levels of R&D investments and outputs only in limited cases as in competitive ($\beta = 0$) and collusive ($\beta = 1$) equilibria. However, by the new approach utilizing the character of CVs in R&D that their value corresponds to the degree of R&D competition or collusion, we have demonstrated that the same results as their results also hold in a less collusive ($0 \le \beta \le 1$) case, not limited to the full collusive case. Furthermore, we have shed light on the various cases where firms possess negative CVs, namely become involved in both Bertrand competition and intermediate competition between Bertrand and Cournot competition.

Finally, the use of CVs has the merits of enabling us to visually depict the relationship between competition (or collusion) and firm decision.

[14] d' Aspremont and Jacquimin (1998) and Suzumura (1992) do not compare welfare under several market structures. Qiu (1997) compares welfare when firms are involved in Bertrand and Cournot competition in the second stage of quantity decision, but in competition in the first stage of R&D decision.

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